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Analysis of Compliance Maps with MATLAB Toolbox

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Abstract:

Structural dynamics engineers are asked to make measurements for distinguishing differences between two structures. A frequent approach is to compare the modal parameters by change in Modal Assurance Criterion behaviour which focuses only on the Eigen values and Eigen vectors.

A Graphical method of comparison was found for simple structures, which considers the spatial distribution of the entire frequency response spectrum. Point compliances represented by a color scale are measured at short intervals along reference line on a structural edge and the result is a three dimensional representation of dynamic behaviour. Experiments were conducted and MATLAB toolbox along with Graphical User Interface for compliance maps was built.

Keywords:

Experimental Modal Analysis, Frequency Response Functions, Compliance Maps, Dynamic Frequency Mobilyzer, accelerometer, MATLAB.

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Karlskrona, April 2005.

Haribabu Gorrepati.

Chinthalapalle Chandrasekhar Reddy.

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1 Notation

$A(w)$	Accelerance (m/sec^2)
c	Damping ($N \text{ sec}/m$)
$[C]$	Damping matrix (Ns/m)
f	Force (N)
F	Frequency (Hz)
$\{F\}$	Force vector or input signal (N)
$[H]$	Transfer function matrix (rad/sec)
$[H(w)]$	Frequency response function $(-)^*$
k	Spring coefficient (N/m)
$[K]$	Stiffness Matrix (N/m)
m	Mass (kg)
$[M]$	Mass matrix (kg)
N	Number of mode of interest $(DL)^*$
s	Laplace operator $(DL)^*$
t	Time variable (sec)
x	Displacement (m)
$\{x\}$	Time varying displacement vector
$\{X\}$	Response vector or out put signal $(-)^*$
$\dot{x}$	First derivative with respect to time of dependent variable x
$\ddot{x}$	Second derivative with respect to time of dependent variable
$\{\dot{x}\}$	Time varying velocity vector
$\{\ddot{x}\}$	Time varying acceleration vector
$V(w)$	Mobility (m/sec)
w	Angular frequency

Indices

0	Initial conditions
i	i th Degree of freedom
j	j th Degree of freedom
k	k th Degree of freedom
x	X-direction
y	Y-direction
z	Z-direction

Abbreviations

SDOF	Single Degree of Freedom
MDOF	Multi Degree of Freedom
FFT	Fast Fourier Transform
EMA	Experimental Modal Analysis
DFM	Dynamic Frequency Mobilyzer
UFF	Universal File Format
TPS	Traction Power System

2 Introduction

Vibrations are inherent to life though generally mankind regards them as unpleasant causing undesirable consequences as discomfort, noise, malfunctioning, wear, fatigue and even destruction.

Therefore strong and reliable vibration analysis tools are a basic need of modern engineering. Modal analysis is one of those tools. There are many methods to distinguish differences between two structures within modal analysis, but the draw back with all these methods is that these cannot give you the visualization of the above said differences.

Background

A collection of a series of high quality, closely spaced FRFs over some length or area can be used to visualize and show for efficiently comparing the dynamic properties of structures. This visualization is termed Compliance Maps [1]. Work has been carried out in this field by Gary C. Foss [1], where Compliance maps were drawn for trusses, Snowboards, Wing Spar mill fixtures etc.

Aim and Objective

This thesis is an attempt to extend work on this new visualization tool to mark the differences between two structures. Compliance maps were drawn for various skis. A MATLAB program with a graphical user interface was designed for plotting compliance maps.

The main objectives of the thesis can be listed as

- Study of various methods for comparison of Structures
- Simulation of rectangular plate in MATLAB followed by Experimentation on same in the Laboratory, for acquaintance in Modal parameter extraction, Water fall diagrams and compliance maps.
- Extension of the experimentation on Skis of different ages and plotting the compliance maps.
- Development of a MATLAB toolbox along with a GUI to enable plotting of compliance maps for any given data.

This report explains the various theories behind Modal parameter extraction, Water fall diagrams and compliance maps followed the description of experimental setup in chapter 4. Results of the experiments are presented in chapter 5 followed by conclusions. The toolbox and GUI are explained in *Appendix A*.

3 Theory of Modal Analysis

Modal analysis is primarily a tool for deriving reliable models to represent the dynamics of structures. Modal analysis aims to develop reliable dynamic models that may be used with confidence in further analysis. In general, it can be said that the applications of modal analysis today cover a broad range of objectives like,

- Identification and evaluation of vibration phenomena
- Development of experimentally based dynamic models
- Structural integrity assessment
- Structural modification and damage detection
- Model integration with other areas of dynamics such as acoustics, fatigue, etc.
- Establishment of criteria and specifications for design, test, qualification and certification

Models of vibrational mechanical systems consider mainly masses, stiffness and damping. Every model uses assumptions for simplification and therefore contains uncertainties from the outset. Because of the increasing importance of a preferable precise estimation of the model parameters, the computer aided measurement and analysis of dynamical properties of components play a more and more decisive role. A structure's actual dynamical behaviour can merely be investigated experimentally. The Experimental Modal Analysis (EMA) is one of the most important measurement procedures in this area.

The EMA uses transfer functions of a system, i.e. the relationship between 'system input' (driving forces) and 'measured system response signals' (accelerations at one or more points on the structure). The measured time domain signals are transformed into the frequency domain. Each transfer function gained from a single output-input combination is one component of the overall system. From this the modal quantities Eigen frequencies, damping parameters and eigenvectors (spatial displacements at the points of measurement at the respective Eigen frequency also called mode shapes) can be calculated. The knowledge of the modal quantities allows a

description of the dynamic behavior and is the basis of further numerical investigations.

Modes (or resonances) are inherent properties of a structure. Resonances are determined by the material properties (mass, stiffness, and damping properties), and boundary conditions of the structure. Each mode is defined by a natural (modal or resonant) frequency, modal damping, and a mode shape. If either the material properties or the boundary conditions of a structure change, its modes will change. At or near the natural frequency of a mode, the overall vibration shape (operating deflection shape) of a structure will tend to be dominated by the mode shape of the resonance [2].

3.1 Multiple Degrees of Freedom System

The degrees of freedom of a system are the number of independent coordinates necessary to completely describe the motion of that system.

The simplest possible discretisation of a system, denoted as a SDOF (Single Degree of Freedom), was introduced as a model capable of describing its dynamic behaviour in the simplest possible terms. The advantage of this initial approach is that it makes it a lot easier to understand most of the basic concepts and their physical meaning. However, most real mechanical systems and structures cannot be modelled successfully by assuming a single degree-of-freedom, i.e. single coordinate to describe their vibratory motion.

Real structures are continuous and non homogeneous elastic systems which have an infinite number of degrees of freedom. Therefore their analysis always entails an approximation which consists of describing their behaviour through the use of a finite number of degrees of freedom, as many as necessary to ensure enough accuracy.

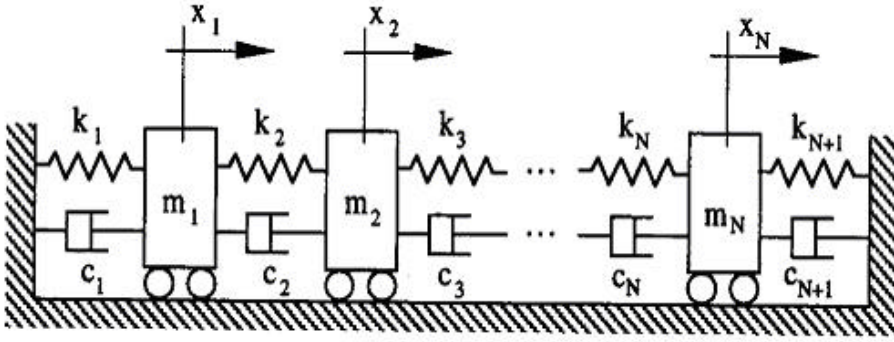


Figure 3.1. Example of a model with 'N' degrees of freedom.

Figure 3.1 representing a viscously damped system described by its spatial mass, stiffness and damping properties. A total of N coordinates $x_i(t)$ ($i = 1, 2, 3, \dots, N$) are required to describe the position of the N masses relative to their static equilibrium positions and the system is said to have N degrees of freedom.

Assuming that each mass may be forced to move by an external force $f_i(t)$ ($i = 1, 2, 3, \dots, N$) and establishing the equilibrium of the forces acting on them.

The motion of the system of equations for the given system is,

$$m_1 \ddot{x}_1 + (c_1 + c_2) \dot{x}_1 - c_2 \dot{x}_2 + (k_1 + k_2) x_1 - k_2 x_2 = f_1 \quad (3.1)$$

$$m_2 \ddot{x}_2 - c_2 \dot{x}_1 + (c_2 + c_3) \dot{x}_2 - c_3 \dot{x}_3 - k_2 x_1 + (k_2 + k_3) x_2 - k_3 x_3 = f_2 \quad (3.2)$$

$$m_3 \ddot{x}_3 - c_3 \dot{x}_2 + (c_3 + c_4) \dot{x}_3 - c_4 \dot{x}_4 - k_3 x_2 + (k_3 + k_4) x_3 - k_4 x_4 = f_3 \quad (3.3)$$

$$m_4 \ddot{x}_4 - c_4 \dot{x}_3 + c_4 \dot{x}_4 - k_4 x_3 + k_4 x_4 = f_4 \quad (3.4)$$

$$m_N \ddot{x}_N - c_N \dot{x}_{N-1} + (c_N + c_{N+1}) \dot{x}_N - k_N x_{N-1} + (k_N + k_{N+1}) x_N = f_N \quad (3.5)$$

A convenient method to solve the above system of equations is to use matrices.

From the above equations we can write the Mass matrix as

$$M = \begin{bmatrix} m_1 & 0 & \cdot & \cdot & 0 \\ 0 & m_2 & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & m_N \end{bmatrix}$$

Damping matrix as

$$C = \begin{bmatrix} c_1 + c_2 & -c_2 & \cdot & \cdot & 0 \\ -c_2 & c_2 + c_3 & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & c_N + c_{N+1} \end{bmatrix}$$

And Stiffness matrix is

$$K = \begin{bmatrix} k_1 + k_2 & -k_2 & \cdot & \cdot & 0 \\ -k_2 & k_2 + k_3 & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & k_N + k_{N+1} \end{bmatrix}$$

For a multiple degrees of freedom system, the system of equations can be written as in the form of matrix notations is,

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{f\} \quad (3.6)$$

Where $[M]$, $[C]$, and $[K]$ are Mass, Damping and Stiffness symmetric matrices, these matrices describes the spatial properties of the system. $\{\ddot{x}\}$, $\{\dot{x}\}$ and $\{x\}$ vectors are time varying acceleration, velocity and displacement responses, and $\{F\}$ is the time varying external excitation forces.

By taking Laplace transform on both sides of the above equation we will get,

$$L([M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\}) = L(\{f\}) \quad (3.7)$$

$$(Ms^2 + Cs + k).X(s) = F \quad (3.8)$$

Equations (3.1) to (3.5) are second order, linear and time invariant differential equations. Since the system is coupled it must be solved simultaneously. By assuming that the system is un-damped the solution is obtained. Since the system is coupled it has to be manipulated to form an Eigen value problem. The resulting Eigen value represents the frequencies of the modes and the eigenvectors are the mode shapes.

3.2 Frequency Response Measurements

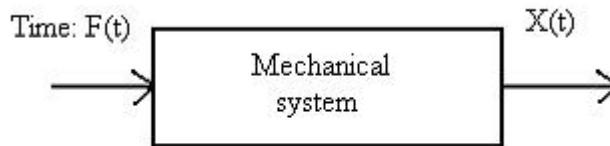
The frequency Response Function (FRF) is a fundamental measurement that isolates the inherent dynamic properties of a mechanical structure. Experimental Modal Parameters (Frequency, damping and mode shape) are also obtained from a set of FRF measurements.

The FRF describes the input-output relationship between two points on a structure as a function of frequency. Since both force and motion are vector quantities, they have directions associated with them. Therefore, an FRF is

actually defined between a single input DOF (point & direction), and a single output DOF.

Frequency Response Function is a measure of how much displacement, velocity or acceleration response a structure has at an output DOF per unit of excitation force at an input DOF as a function of frequency.

FRF is the ratio of the Fourier transform of an out put response $X(\omega)$ to the Fourier transform of input force $F(\omega)$.



$$\text{Frequency } F(\omega) * [H(\omega)] = X(\omega)$$

Figure 3.2. Definition of FRF.

3.2.1 Types of FRFs

Motion can also be expressed as acceleration or velocity. For modal analysis, the most common form of the FRF is the normalized acceleration response or acceleration.

$$A(\omega) = \frac{A}{F} \tag{3.9}$$

The inverse of acceleration is called the apparent mass. The velocity FRF is called the mobility.

$$V(\omega) = \frac{V}{F} \tag{3.10}$$

The inverse of mobility is called the mechanical impedance. The displacement response function $H(w)$ is called the dynamic compliance or receptance and the inverse of this is called the dynamic stiffness.

$$H(w) = \frac{X}{F} \quad (3.11)$$

The choice of displacement, velocity or acceleration depends on the phenomena of interest. If amplitude and frequency are logarithmically scaled, the choice will affect the average slope of the FRF and hence scaling relative to frequency. Since the measurements are typically made with an accelerometer, acceleration and apparent mass are the easiest quantities to display. However, displacement, stiffness and compliance are more useful and intuitive quantities for comparing structures whose performance depends on resistance to displacement.

3.2.2 The FRF Matrix Model

A structural dynamics measurement involves measuring elements of an FRF matrix model for the structure. This model represents the dynamics of the structure between all pairs of input and output DOF's. The FRF matrix model is a frequency domain representation of a structure's linear dynamics, where linear spectra (FFT's) of multiple inputs are multiplied by elements of the FRF matrix to yield linear spectra of multiple outputs.

FRF matrix columns correspond to inputs and rows correspond to outputs. Each input and output corresponds to a measurement DOF of the test structure. The FRF measurements on a structure are shown in *figure 3.3*.

3.3 Modal Testing

In modal testing, FRF measurements are usually made under controlled conditions, where the test structure is artificially excited by using either an impact hammer, or one or more shakers driven by broadband signals. A multi-channel FFT analyzer is then used to make FRF measurements between input and output DOF pairs on the test structure. Here for the SKI structure we used impact hammer method.

When the output is fixed and FRFs are measured for multiple inputs, this corresponds to measuring elements from a single row of the FRF matrix. This is typical of a roving hammer impact test.

$$\begin{Bmatrix} x_1(j\omega) \\ x_2(j\omega) \\ x_3(j\omega) \\ \dots \\ \dots \end{Bmatrix} = \begin{bmatrix} \dots & \dots & h_{17}(j\omega) & \dots & \dots \\ \dots & \dots & h_{27}(j\omega) & \dots & \dots \\ \dots & \dots & h_{37}(j\omega) & \dots & \dots \\ \dots & \dots & h_{47}(j\omega) & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \end{bmatrix} \begin{Bmatrix} \dots \\ \dots \\ f_7(j\omega) \\ \dots \\ \dots \end{Bmatrix}$$

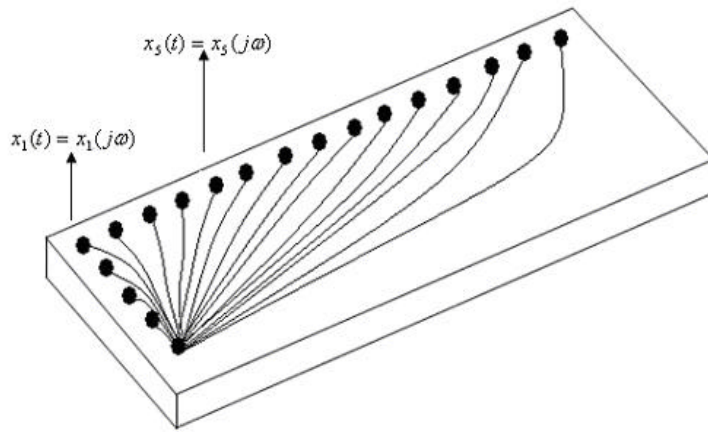


Figure 3.3. Measurements of FRFs on a structure.

3.3.1 Measuring FRF Matrix Rows or Columns

Modal testing requires that FRFs be measured from at least one row or column of the FRF matrix. Modal frequency and damping are global properties of a structure, and can be estimated from any or all of the FRFs in a row or column of the FRF matrix. On the other hand, each mode shape is obtained by assembling together FRF numerator terms (called residues) from at least one row or column of the FRF matrix.

3.3.2 Exciting Modes with Impact Testing

Impact testing is the most popular modal testing method used today. It is a fast, convenient and low cost way of finding modes of the structure. The test equipment to perform the operation is an impact hammer with a load cell attached to its head to measure the input force, an accelerometer to measure the response acceleration at a fixed point and direction, a two or four channel FFT analyzer to compute FRFs.

An example of time signal of input and response and their spectrums are shown in the *Figure 3.4a* and *Figure 3.4b* respectively.

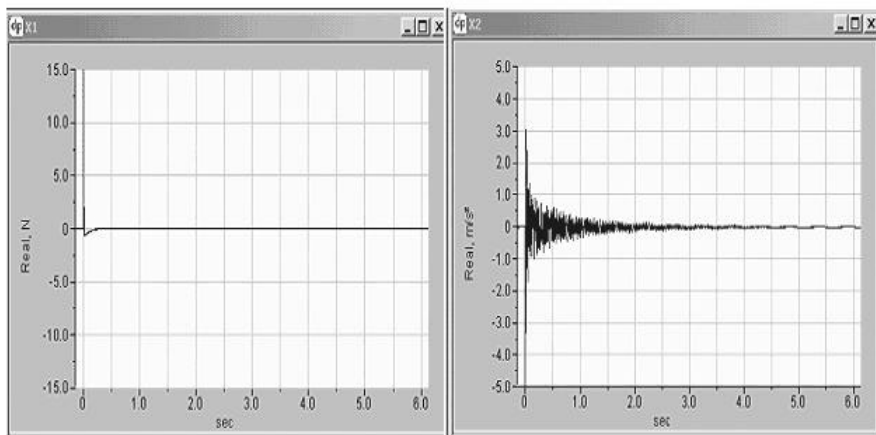


Figure 3.4(a). Impulse and Response signals (Time Domain).

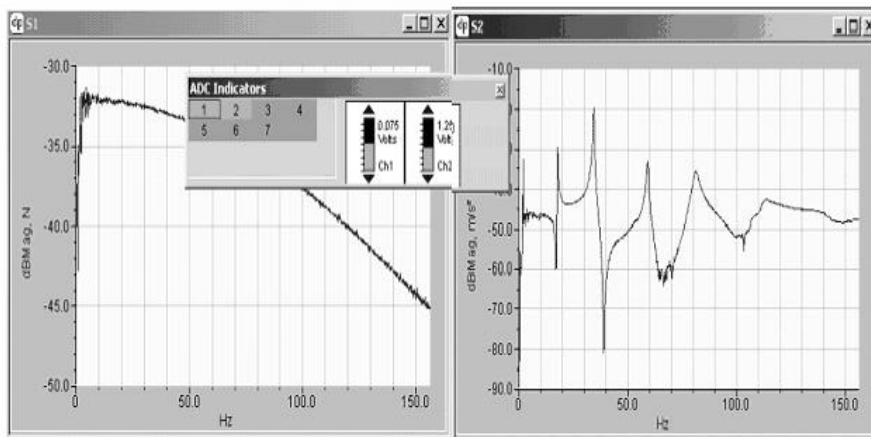


Figure 3.4(b). Impulse and Response Spectrums.

3.4 Data Acquisition

The used data acquisition system is of type SignalCalc Mobilyzer® manufactured by Data Physics ® Corporation. The setup of the Mobilyzer® is shown in *Figure 3.5*

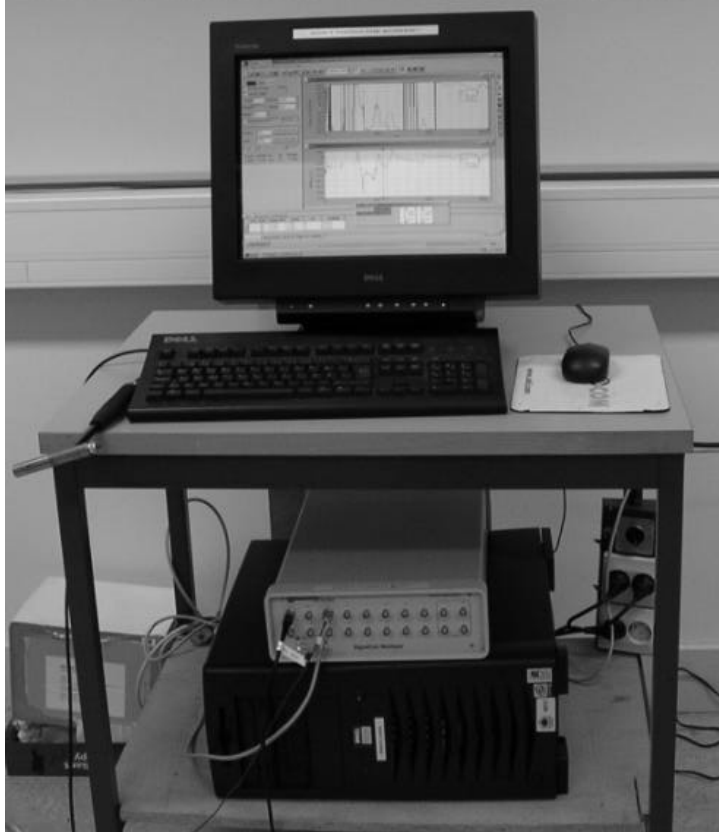


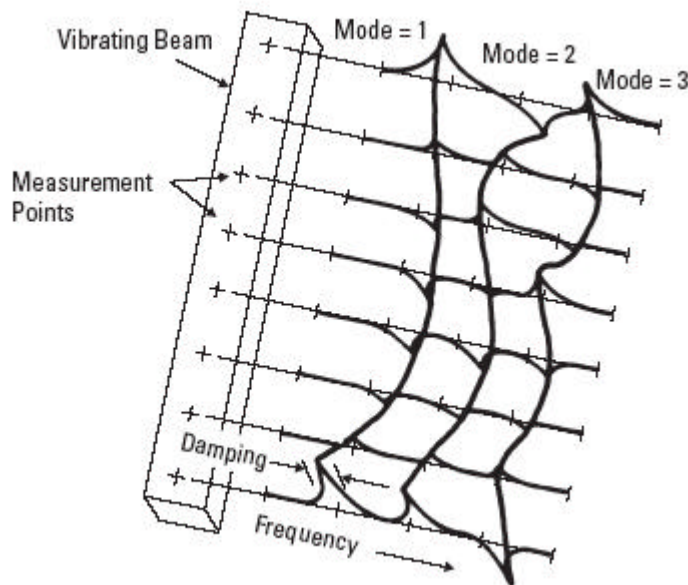
Figure 3.5. SignalCalc Mobilyzer® Setup.

3.5 Modal Parameter Extraction

One of the most fundamental assumptions of modal testing is that a mode of vibration can be excited at any point on the structure, except at nodes of vibration where it has no motion. Hence, a single row or column of the

frequency response matrix provides sufficient information to estimate modal parameters. As a result, the frequency and damping of any mode in a structure are constants that can be estimated from any one of the measurements as shown in *Figure 3.6*. In other words, the frequency and damping of any mode are global properties of the structure.

In practical applications, it is important to include sufficient points in the test to completely describe all the modes of interest [11]. If the excitation point has not been chosen carefully or if enough response points are not measured, then a particular mode may not be adequately represented. At times it may become necessary to include more than one excitation location in order to adequately describe all of the modes of interest. Frequency responses can be measured independently with single-point excitation or simultaneously with multiple-point excitations. The mode shapes as a whole are also global properties of the structure, but have relative values depending on the point of excitation and scaling and sorting factors. On the other hand, each individual modal coefficient that makes up the mode shape is a local property in the sense that it is estimated from the particular measurement associated with that point as shown in *figure 3.6*.



Damping frequency – Same at each Measurement Point

Mode Shape – Obtained at same frequency from all measurement points

Figure 3.6. Concepts of modal parameters.

3.6 Definitions of Waterfall Diagrams and compliance maps

3.6.1 Waterfall diagrams

Waterfall diagram is a 3-dimensional representation with frequency on the x -axis amplitude on the y -axis and nodes of the structure on the z -axis. *Figure3.7* shows a mobility waterfall diagram created on a simulated rectangular plate in MATLAB®.

3.6.2 Compliance maps

Compliance maps are color diagrams with frequency on the x -axis physical position on y -axis and color scale represents amplitude.

The FRF magnitudes should usually be scaled logarithmically. This displays the FRF peaks and dips with equal symmetry. For compliance color-maps, this means opposite colours equally represent extremes of stiffness or compliance and deep blue would represent large apparent mass [1].

Viewing data in this way gives a depiction of how the dynamic properties distribute themselves both spectrally and spatially. The peaks of acceleration line up at constant frequency and vary only in amplitude, defining the mode shape along the reference line. For drive point FRFs, every resonance is followed by anti resonance. The anti-resonance do not line up at a single frequency. Their frequencies wander with position from the node line at one mode to node line at next mode.

3.6.3 Waterfall diagrams Vs Compliance maps

As an alternative to the waterfall diagram, the data can be displayed as a color map shown in the *figure3.7*. The color scale now represents amplitude and the Y direction represents physical position. This type of three-dimensional display improves on the waterfall diagram for conceptualizing the spatial as well as spectral nature of structural dynamic behaviour. The modes show up as vertical bars of high acceleration (yellow and or red). The anti-resonance areas are distinguished as dark blue or green valleys of high

apparent mass. These features are not distorted, as they are in the waterfall display. As used here, the term anti-resonance refers to a local minimum in an FRF. A node would be a location where anti-resonance intersects with a mode frequency, indicating no motion at that location for the mode.

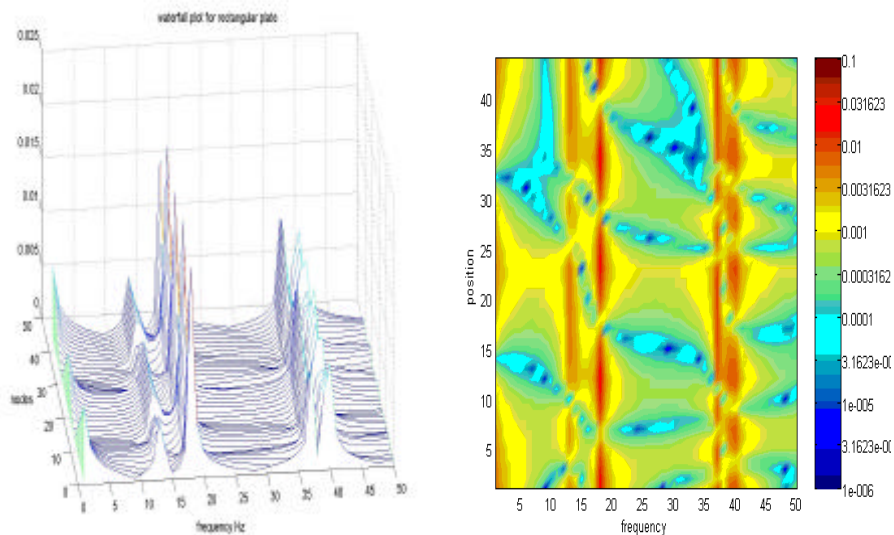


Figure 3.7. Water fall Diagram vs. Compliance Maps.

4 Experimentation

4.1 Simulated Rectangular plate

A rectangular plate of $2000\text{mm} \times 1000\text{mm} \times 100\text{mm}$ was simulated in MATLAB® as shown in *figure 4.1* with 12 nodes on each side with a Young's Modulus of $2.1 \times 10^{11} \text{ N/m}^2$, and density 7850 kg/m^3 . Various experiments on the simulated plate were conducted with the help of MATLAB® functions provided by Saven Edutech® and the results in the form of FRF curves, waterfall diagrams and compliance maps are incorporated in chapter 5.

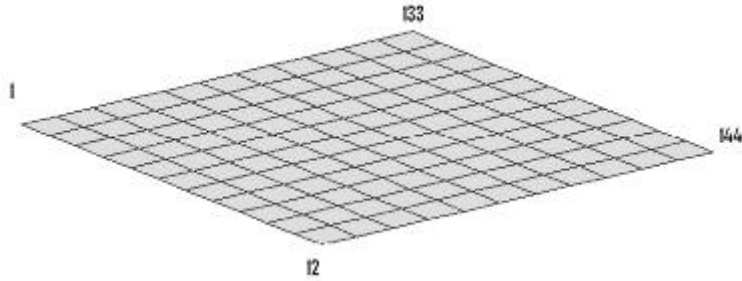


Figure 4.1. Simulated Rectangular plate in MATLAB.

4.2 Rectangular plate

A rectangular plate of $990\text{mm} \times 128\text{mm} \times 30\text{mm}$ was taken and the experiments were performed by attaching mass tuned damper as shown in *Figure 4.2*. The construction of mass tuned damper was made with the help of sponge and Swedish coins glued together. The resonance frequency of the mass tuned damper an SDOF system is changed by reducing or increasing the number of the coins accordingly.

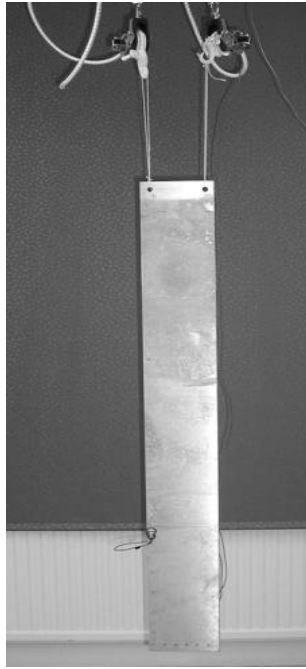


Figure 4.2. Rectangular Plate hanged with free-free conditions.

4.3 Skis

Three different types of skis were selected as shown in *Figure 4.3* and compliance maps are plotted to show the differences between these three different skis. The results in the form of compliance maps are included in chapter 5.

From the *Figure 4.3* the yellow colored ski is obsolete, the white one is in use and the red colored ski is new and supplied by ‘HEAD®’ – a leading sports gear manufacturing company in the world.

The white and the red skis came with TPS (Traction Power System) as shown in *Figure 4.4* and the manuals which accompanied the skis indicated that the TPS aids in making wide and tight turns in different ski conditions extremely smooth. Initially the TPS was tested for its operations although the main aim remained as to represent the modal parameters in the form of a compliance map.

The fixtures and bindings on the white and the yellow skis act like point masses attenuating some of the higher resonance frequencies.



Figure 4.3. Shows three different types of skis which were used for experimentation work.



Figure 4.4. Shows the TPS on red and white skis.

4.4 Measurement preparations

The preparations for the measurements have great significance to the quality of the collected data. The measurements should be done according to the following steps.

- Suspension of the structure
- Selection of excitation point
- Method to excite the structure
- Selection of measurement points
- Accelerometer considerations

4.4.1 Suspension of the structure

There are two different ways to suspend the structure when performing experimental modal analysis.

- Free –free conditions
- Operating conditions

A free-free condition is the best method to measure since the boundary conditions are easy to achieve in a repeatable way. In this the energy is mainly spread into the structure not into the surrounding parts. *Figure 4.5* shows the free –free conditions.

The second way of suspension is in its operating conditions in this case there are two possible ways. This can be achieved by experimenting directly while in operating conditions or approximated in the laboratory.



Figure 4.5. Ski hanged with free-free conditions.

4.4.2 Selection of excitation points

The number and placement of the exciters should be chosen so that all the modes of interest are excited properly. So the excitation point must not be located near a nodal point for any mode [3].

Without prior knowledge of the dynamic characteristics of a structure the location of excitation measurement points is a matter of trial and error coupled with experience and engineering judgment.

Generally it is recommended to choose one corner of the structure as excitation point but in order to select a proper reference point a simple FE model of is recommended to use, if one is available, otherwise this can be done by experimental investigation of several possible points by measuring

the driving point frequency response. This can be done with the use of a single accelerometer and a roving impact hammer.

4.4.3 Method to excite the structure

The excitation mechanism is constituted by a system which provides the input motion to the structure under analysis. There are two types of excitation methods possible for measurement of FRFs for modal analysis of a structure.

- Impact hammer excitation
- Shaker excitation

Impact hammer excitation is the best choice to excite small structures. An impact hammer is simply a hammer with various attachable masses and tips which serve to extend the frequency and force ranges of the impact as shown in *figure 4.6*. An integral part of the hammer is a force transducer, which uses the compression of a piezoelectric crystal to detect the magnitude of the force felt by the hammer when it strikes a structure. The magnitude of the force is determined by the mass of the hammer head and the velocity with which it is moving when it hits the structure. When operated by hand, it is usually easier to vary the velocity, so the force level may be adjusted by changing the mass of the hammer head. The frequency range of excitation provided by a hammer is determined by the stiffness of the hammer-structure contact surfaces and the mass of the hammer head.

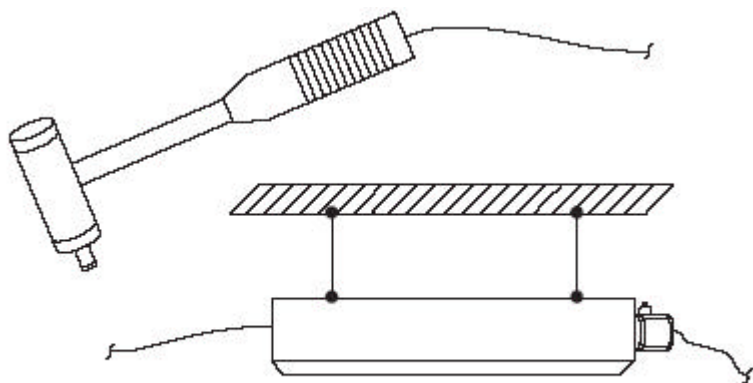


Figure 4.6. An impact hammer which is used to excite the structure.

The main advantage of impact hammer testing is that the excitation equipment is small, light and cheap. It is mainly used for diagnostic purposes rather than for precise measurement of FRF properties.

Disadvantages of the hammer testing method are related to the inconsistency of the excitation. The impact pulse is difficult to control accurately in size, in shape and in direction, and the duration of the pulse is very small compared with the measurement time frame. It is very important to avoid double hits which result when the hammer bounces against the surface. Double hits cause significant signal processing problems and contaminate measured data.

4.4.4 Method to excite the structure

The choice of response measurement locations should allow unique geometrical description of the mode shapes, avoiding problems of spatial aliasing [2]. The response points are often selected to give a nice display of mode shapes. A response point at the bottom of the ski was selected as shown in *Figure 4.7*.



Figure 4.7. Selection of response point at the bottom of the ski.

4.4.5 Accelerometer considerations

4.4.5.1 Type of accelerometer

We used the 8772A5M10 accelerometer in our experiment. This accelerometer can operate both as standard low impedance, voltage mode sensor with a conventional analog output signal or in a digital piezo smart sensor mode capable of providing pertinent information stored within its memory module. This type of accelerometer is ideally suited for multi channel modal analysis applications. The convenient cubic configuration provides flexibility for installation. Any of three orthogonal surfaces can be used for adhesive attachment, allowing quick removal and convenient orientation alignment. Main applications are for multi channel measurements, Modal analysis measurements on automotive body and aircraft frames and structural analysis measurements.

4.4.5.2 Mounting techniques

There are several mounting techniques to mount the accelerometer to the structure. The most common types of mounting techniques are adhesive mount, standard stud mount, magnetic mount and handheld or probe tip mount. Here in our experiment we used adhesive mount, this method involves attaching a base to the test structure, then securing the sensor to the base, it is often used for temporary installation or when the test object surface cannot be adequately prepared for stud mounting. Adhesives like hot glue and wax well work for temporary mounts. Smooth surfaces and stiff adhesives provide the best frequency response.

The connection between accelerometer and structure should be as rigid as possible, due to that mounted resonance frequency is decreased with increasing flexibility.

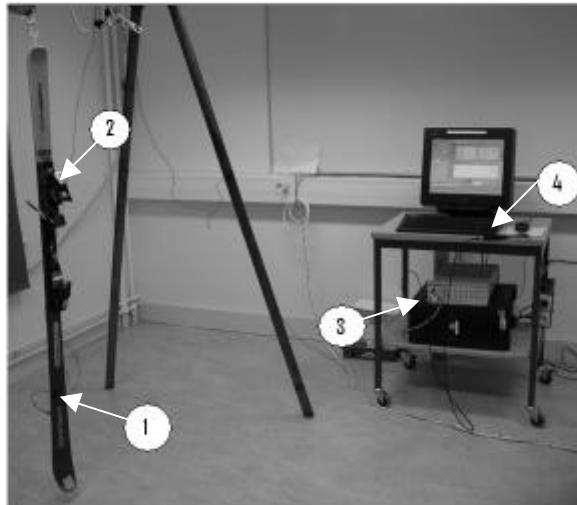
4.4.5.3 Accelerometer calibration

Accelerometer calibration provides, with a definable degree of accuracy, the necessary link between the physical quantity being measured and the electrical signal generated by the sensor. In addition to that other useful information concerning operational limits, physical parameters, electrical characteristics and environmental influences may also be determined.

Under normal conditions, piezo electric sensors are extremely stable, and their calibrated performance characteristics do not change over time. The sensor may be temporarily or permanently affected by harsh environments or other unusual conditions that cause the sensor to experience dynamic phenomena outside of its specified operating range. This change manifests itself in a variety of ways, like a shift of the sensor resonance due to a cracked crystal, a temporary loss of low-frequency measuring capability due to a drop in insulation resistance or total failure of the built –in microelectronic circuit due to a high mechanical shock.

4.5 Experimental setup

As shown in the *Figure 4.8* a ski was hanged free-free conditions with fixtures and accelerometer was attached to Dynamic Frequency Mobilyzer® at the input channel 2. The impact hammer was connected to the Dynamic Frequency Mobilyzer® through the cable at the input channel 1. The FRFs were measured by exciting the structure with the help of impact hammer and the response was measured with the help of accelerometer. The FRFs were obtained from DFM in the form of Universal File Format. The DFM was setup to measure three averages at each node. No window was applied as the force signal is a transient signal. Care was taken that the response signal is captured completely by adjusting number of frequency lines in turn, adjusting the time to capture response or vice versa. Once the settings were fixed in the DFM same settings were used for all nodes. Hammer is moved from one node to another to excite the structure.



1) Accelerometer, 2) Fixtures, 3) DFM, 4) Roving Hammer

Figure 4.8. Experimental setup for ski FRF measurement.

4.6 Analysis

The ski was divided into 48 nodes with 16 nodes on each vertical line as shown in *Figure 4.9*. The experimental modal analysis data was measured at 48 different locations by roving hammer method. After obtaining the FRF data in the form of UFF files they were converted into the form of FRF matrix by using the functions supplied by Saven EduTech® and these functions are explained in the chapter MATLAB® tool box described in *Appendix A*.

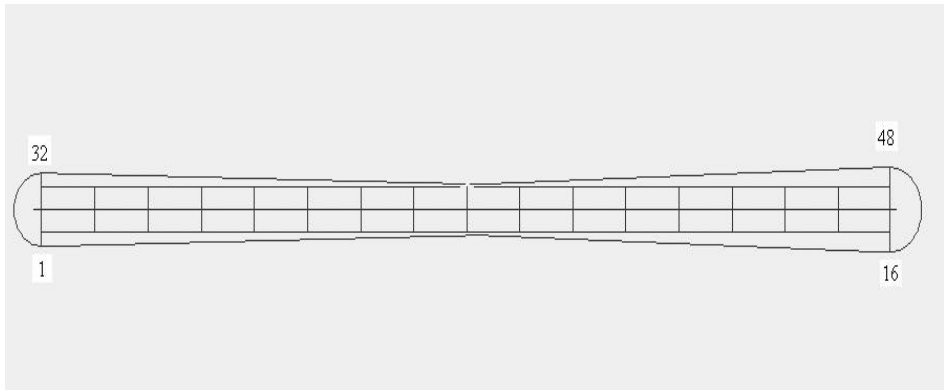
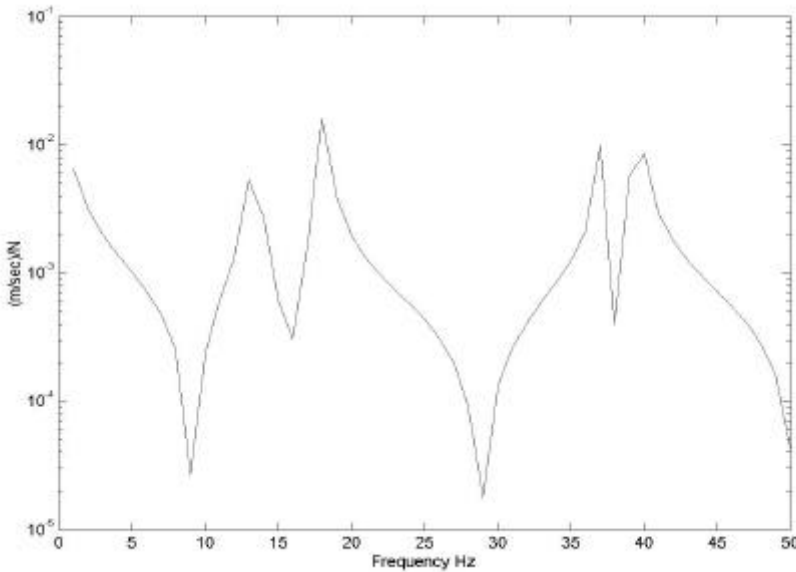


Figure 4.9. Location of nodes on the Ski.

5 Results and Discussion

5.1 Results of Simulated rectangular plate

Firstly, a rectangular plate was simulated and various experiments were conducted using MATLAB®. Plotting frequency response functions before and after attaching mass tuned damper were some of the cases dealt with. The FRFs are expressed in the form of Waterfall diagrams and the same plots are shown in the form of Compliance maps. We have conducted experiments on the rectangular plate in laboratory and compared the results from simulation. This followed experiments on skis and Compliance maps were drawn. In the last phase we made a MATLAB tool box for the analysis of compliance maps along with graphical user interface (GUI). *Figure 5.1* shows the FRF of the simulated rectangular plate.



Fi

gure 5.1. FRF of the simulated rectangular plate.

The resonance frequency at 18.17 Hz has the maximum amplitude. Now with the help of a mass tuned damper we aimed at 18.17 Hz and attenuated

the vibration at that point to half of the original amplitude. The plot which shows the attenuation at 18.17 Hz is shown in *figure 5.2*.

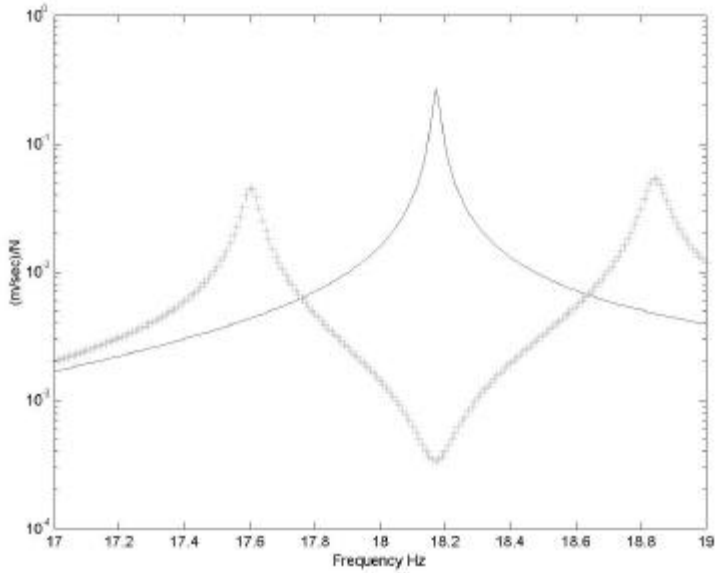


Figure 5.2. Attenuation of aimed resonance frequency for the simulated rectangular plate.

A plot of the waterfall diagram for the above structure (rectangular plate), where the z-axis is nodes along the border of the rectangular plate, is given in *Figure 5.3*.

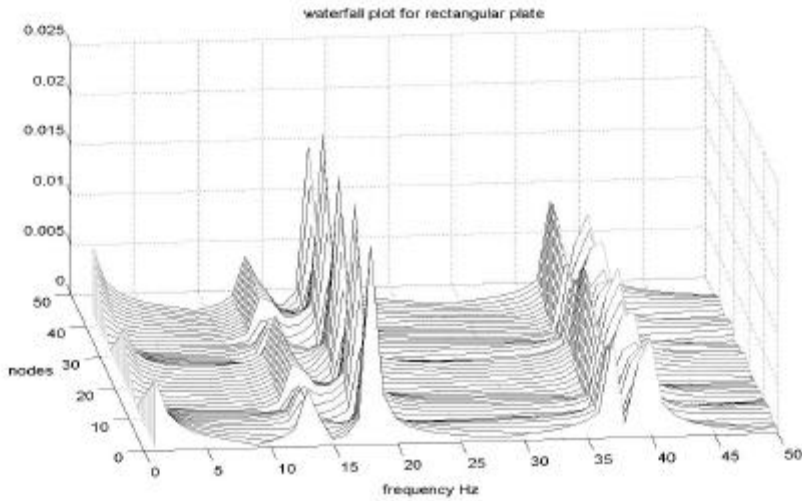


Figure 5.3. Waterfall diagram of simulated rectangular plate without mass tuned damper.

The Water fall diagram after the mass tuned damper was attached to the structure in consideration can be observed in *Figure 5.4*.

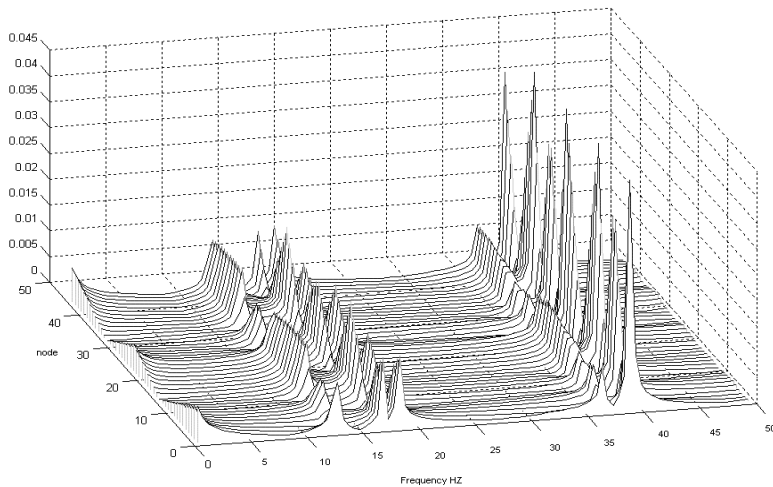


Figure 5.4 Waterfall diagram of simulated rectangular plate after the attachment of mass tuned damper.

The respective compliance maps without damping and with damping are shown in *Figures 5.5* and *Figure 5.6* respectively.

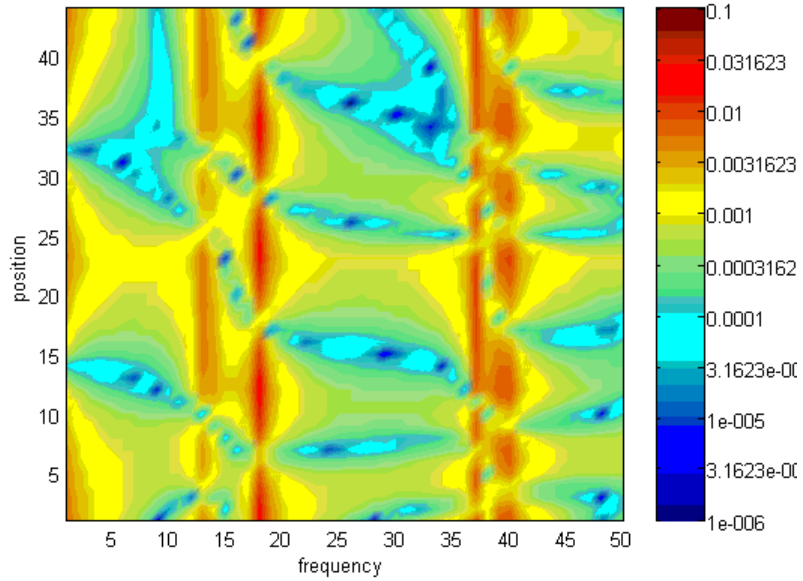


Figure 5.5. Compliance map of simulated rectangular plate without mass tuned damper.

A clear red vertical stripe is observed at about 18.7 Hz as depicted by *Figure 5.5*. This stripe is observed to be split into two vertical stripes (*Figure 5.6*) with a decrease in the color intensity indicating amplitude attenuation which is achieved by the attachment of mass tuned damper. Anti-resonance occurs at the tuned frequency and the vibration is decreased by a large fraction.

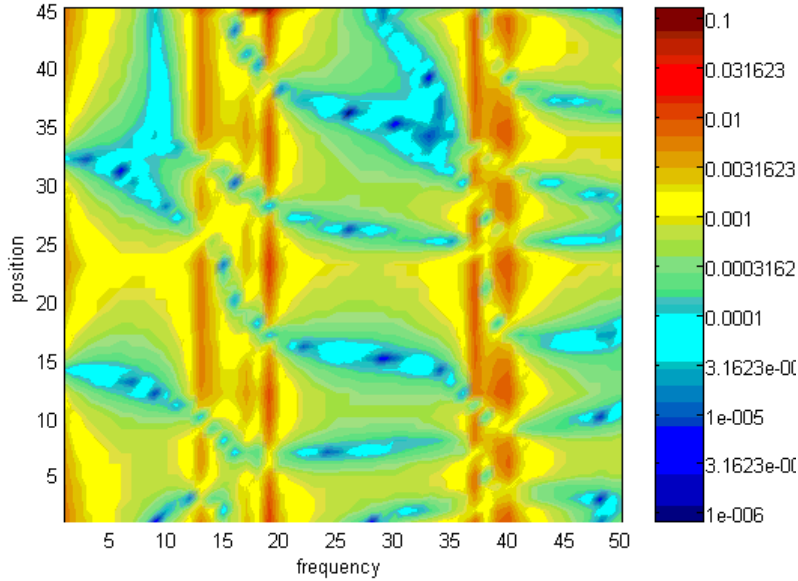


Figure 5.6. Compliance map of simulated rectangular plate after the attachment of mass tuned damper.

5.2 Results of rectangular plate

A rectangular plate of 990mm × 128mm × 30mm was taken and the experiments are performed by attaching mass tuned damper. The Compliance map plotted before attaching the mass tuned damper is shown in *Figure 5.7*.

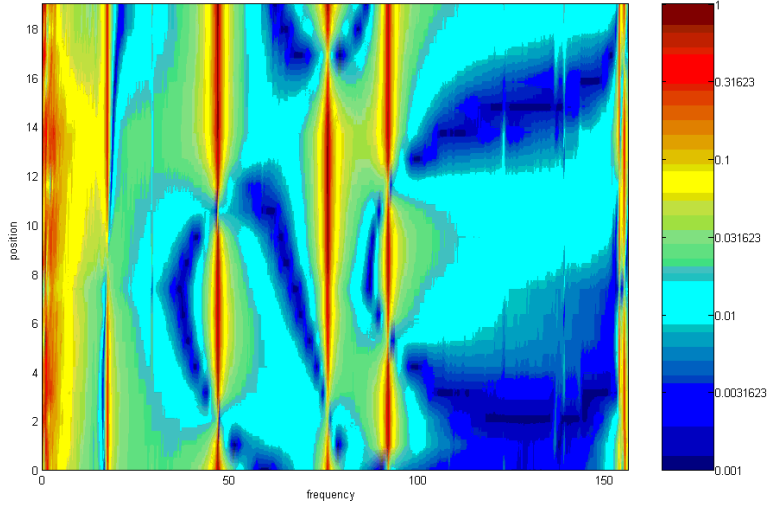


Figure 5.7. Compliance map of rectangular plate before the attachment of mass tuned damper.

Four resonance frequencies at 17, 46, 76 and 92 Hz are observed in *Figure 4.7* and in these resonances mass tuned damper was aimed at 92 Hz. The construction of mass tuned damper was made with the help of sponge and Swedish coins glued together. The resonance frequency of the mass tuned damper which is nothing but an sdof system is changed by reducing or increasing the weight of the coins accordingly. In this way the mass tuned damper was tuned to 92 Hz and the compliance map plotted is shown in *Figure 5.8*.

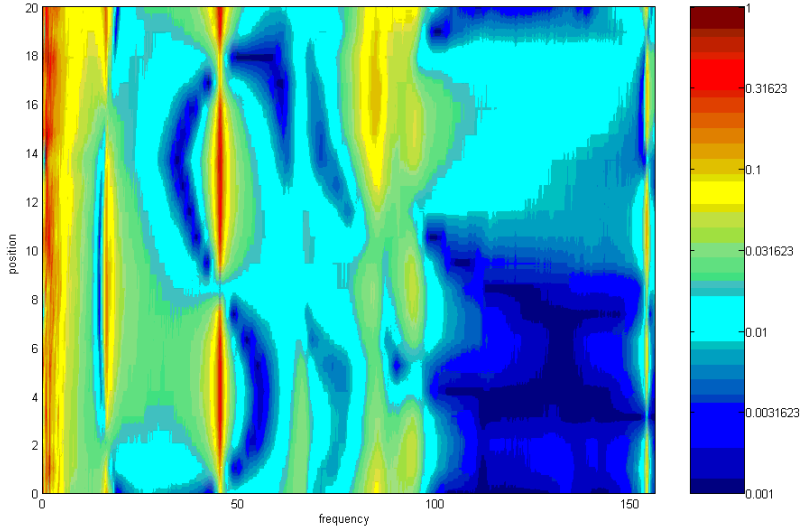


Figure 5.8. Compliance map of simulated rectangular plate after the attachment of mass tuned damper.

It can be observed from the figure that the 92 Hz resonance frequency was attenuated to a large extent. The inaccuracy in the construction of the mass tuned damper led to damping of the unaimed resonance at 76 Hz.

5.3 Results of Skis

The Compliance map of the yellow ski is shown in the *Figure 5.9*

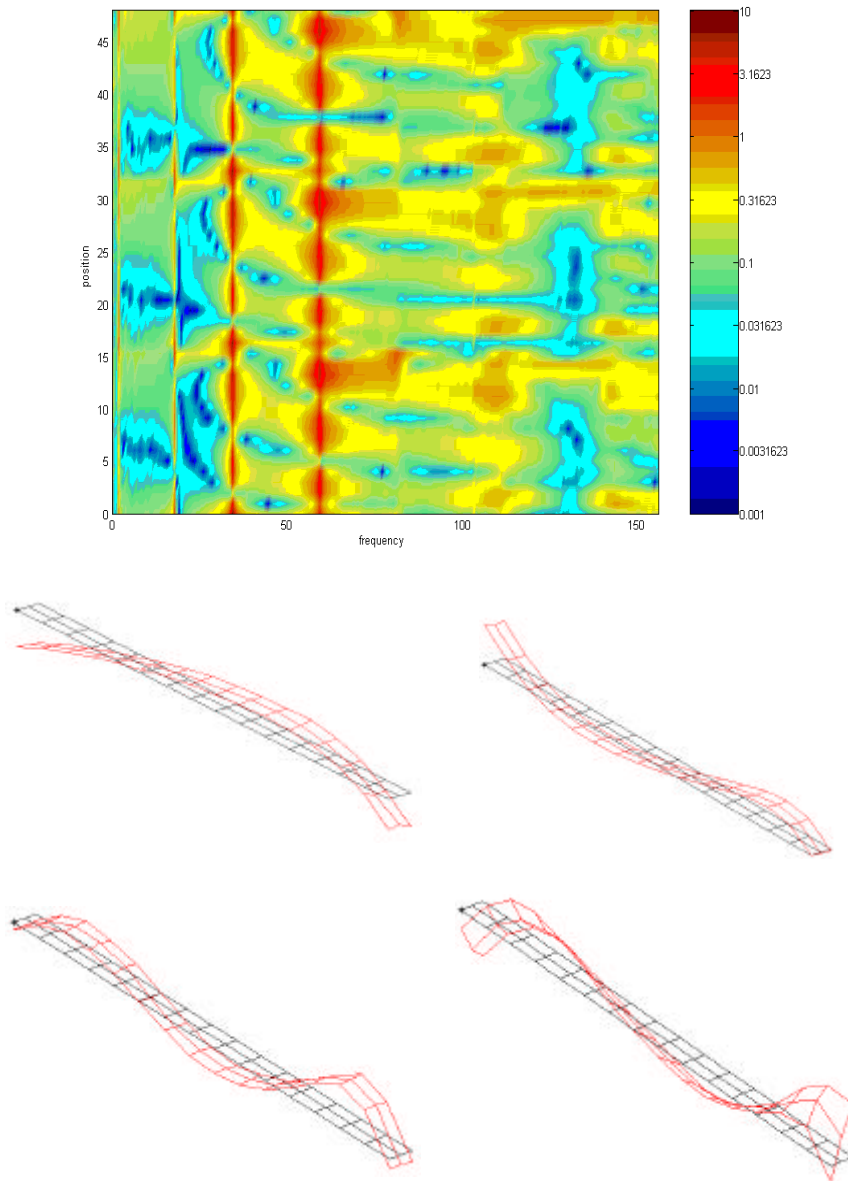


Figure 5.9. Compliance map of the yellow ski and four mode shapes.

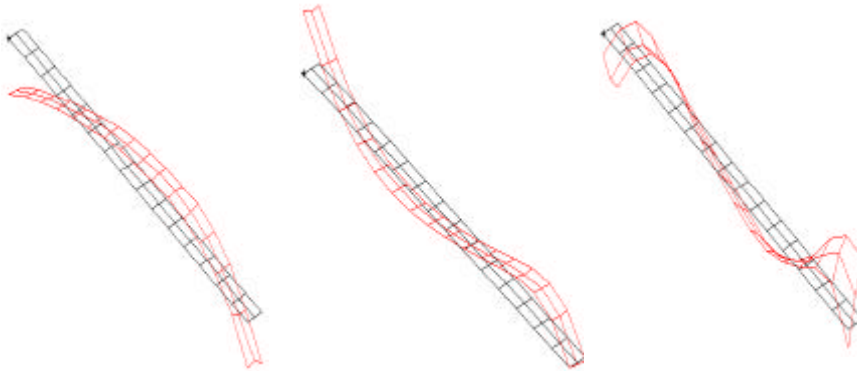
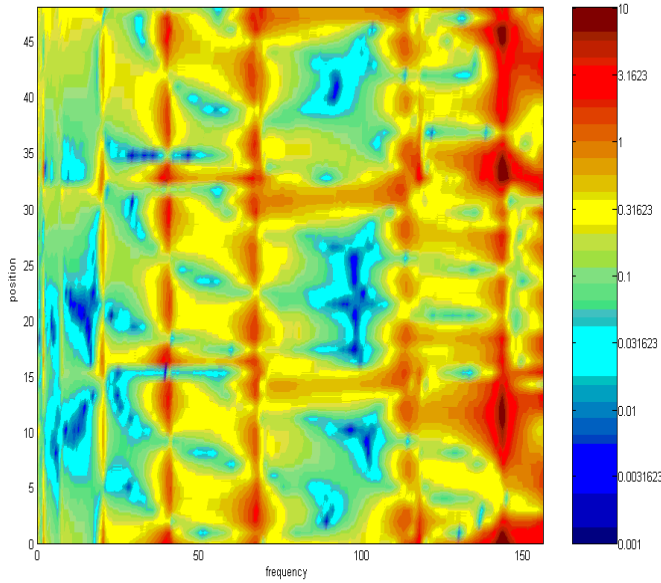


Figure 5.10. Compliance map of the red ski and three mode shapes.

Four resonance frequencies at 1.9, 18, 34.7 and 59.5 Hz are observed in *Figure 5.9* and at dominant 4th resonance frequency torsional mode was observed. The higher modes are damped by the point masses. The first four mode shapes obtained in MATLAB show the First Bending, Second Bending, Third Bending and First torsional mode.

Figure 5.10 shows the compliance map for the red ski, as observed the torsional mode at the 3rd resonance frequency is damped due to the TPS. The first four resonance frequencies are at 19.6, 39.9, 68.1 and 143.9 Hz. The first three mode shapes obtained in MATLAB show the First Bending, Second Bending and First torsional mode.

Figure 5.11 shows the compliance map for the white ski, as observed the torsional mode at the 4th resonance frequency is damped due to the TPS. The first four resonance frequencies are at 2.0, 6.4, 32.5 and 52.6 Hz. The four mode shapes obtained in MATLAB show the First Bending, Second Bending, Third Bending and First torsional mode. There was a good agreement in the scaling of the yellow and red skis but a scaling change is observed in this case. However, the scaling does not affect the working of the TPS. It was observed that the compliance map was inaccurate when ever it was tried to modify to fit along the scale of yellow and red skis.

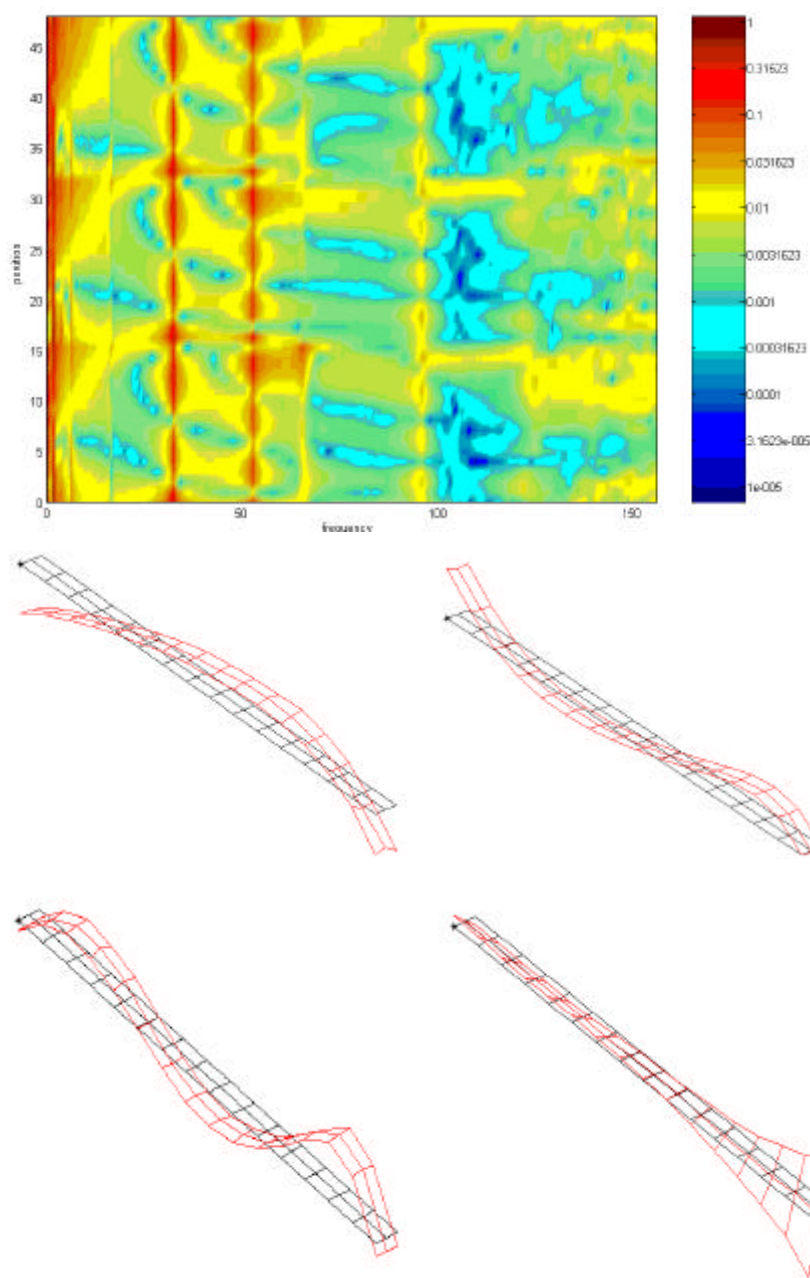


Figure 5.11. Compliance map of the white ski and four mode shapes.

6 Conclusion and Recommendations

Structural dynamics engineers are frequently asked to make measurements for the purpose of distinguishing differences between two structures or changes within the same structure. A frequent approach is to compare the modal parameters by following a shift in modal frequency or a change in modal assurance criterion between two sets of Eigen vectors. But the weakness of this approach is that it ignores the total dynamic behaviour and focuses only on the Eigen values and Eigen vectors.

This thesis is an Extension work of the new visualization tool to mark the differences between two structures. Compliance maps were drawn for various skis. A MATLAB program with a graphical user interface was designed for plotting compliance maps.

A visualization tool has been shown for efficiently comparing the dynamic properties of structures. This technique requires the collection of a series of high quality, closely spaced FRFs over some length or area. It permits quick graphical comparisons, allows the identification of mode shapes for simple structures, and offers additional insight on the spatial and spectral distribution of modal parameters and general dynamic behaviour.

Future work could take into consideration the following points

- More experiments on different simple structures are required. IDEAS model could be built and validated with the help of experimental data presented in this work.
- The MATLAB® tool box can be extended by including more number of functions and look and field of the GUI could be improved by adding more choices for the end user.
- Further work is required on the scaling of amplitude in the form of color intensity in plotting compliance maps.
- Work can be done in studying the comparisons between Compliance maps and other tools of Visualization.

7 References

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- 10 Patrick Marchand and O. Thomas Holland, Graphics and GUIs with MATLAB, 3rd edition, NVIDIA, The Naval Surface Warfare Center Dahlgren Division.
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A User's Manual

1 Introduction

The program is useful to plot compliance maps from universal files obtained by doing experiments on the structures with the help of experimental setup explained in *chapter4.5*. Therefore the knowledge about experimental modal analysis is required and UFF files must be ready to use in this tool box.

A MATLAB® function called '*uicontrol*' is introduced to create the graphic objects in this program, which are user interface controls and they activate call back routines when users activate the objects. There are number of user interface controls like push button, radio button, editable text and menus.

The graphical interface part of the program that is used here was created by use of functions from Saven EduTech®. The functions activate *uicontrol* objects.

After the user had read this manual he will have a good understanding of this MATLAB® program and also will be able to use the program.

2 Window layouts of the GUI

The start screen of the GUI is shown in the *Figure 2.1*. It shows an example plot of Compliance map. In the bottom left corner of the screen a button tagged 'Give Inputs' is provided to enable user to give the required inputs.

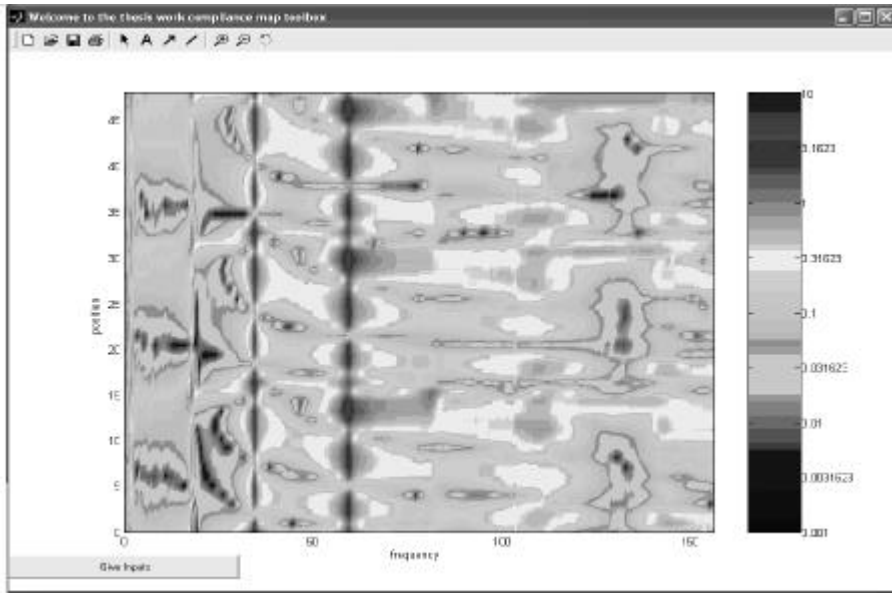


Figure 2.1. Start screen with 'give inputs' button.

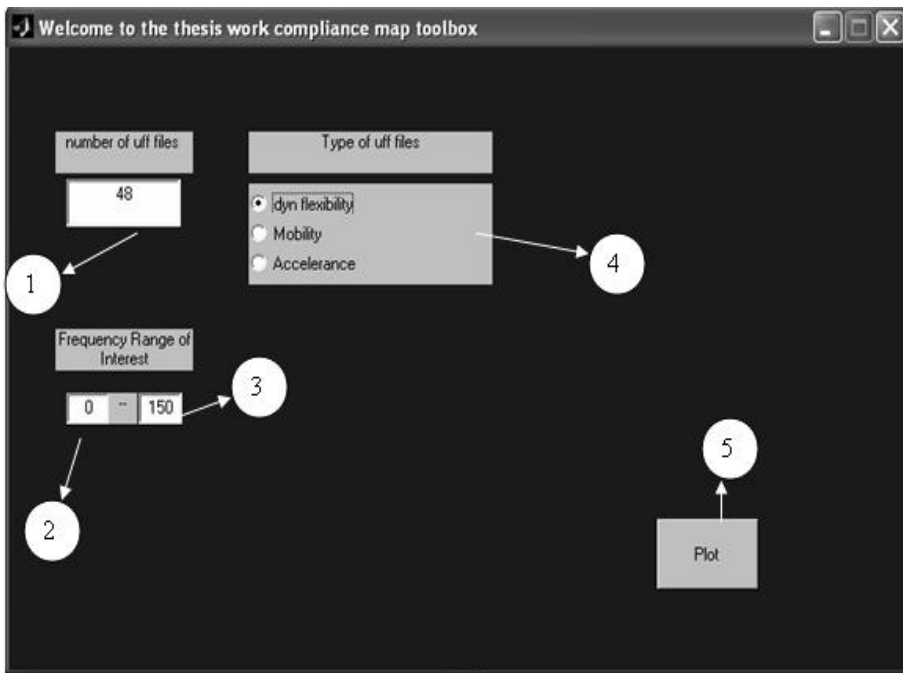


Figure 2.2. User input data window.

As soon as the user hits the ‘Give inputs’ button in the start screen, a new window pops up titled ‘Give Inputs’. It contains 3 edit boxes, 5 static texts, 3 mutually exclusive radio buttons and 1 push button. In *Figure 2.2* the number of UFF files which are used by the user to plot the compliance maps can be entered, the default value in the edit box ‘1000’ (Represented by 1). Frequency range of interest can be entered in 2 and 3. The default value for minimum frequency is ‘0’ and the default value for maximum is ‘78’. User can select the type of UFF files from ‘Dynamic Flexibility’, ‘Mobility’ and ‘Accelerance’ (represented by 4). User can plot the compliance map after the inputs are entered in the respective edit boxes and hitting the ‘Plot’ button (represented by 5).

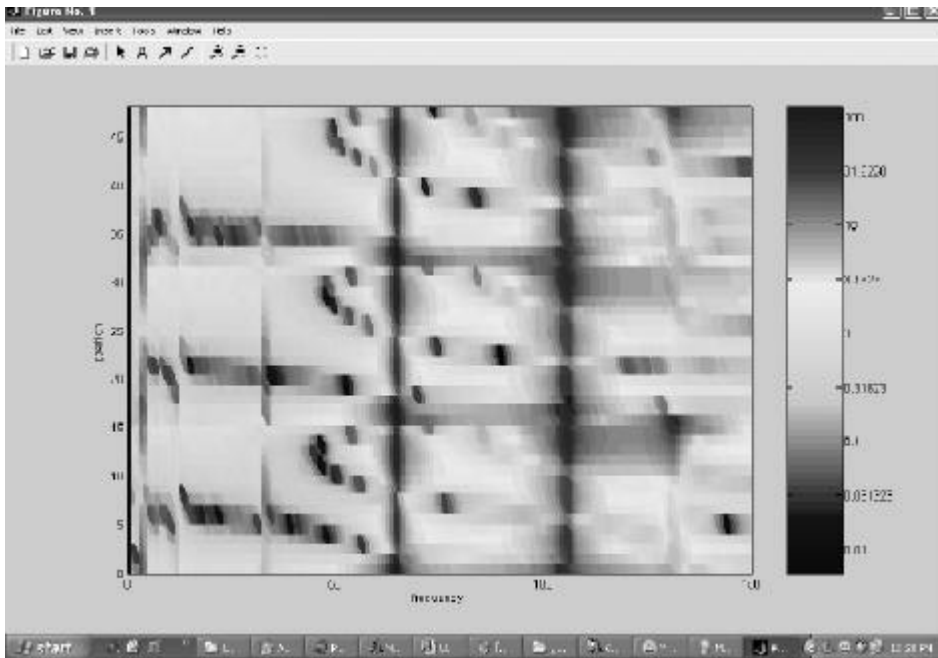


Figure 2.3. Result window with compliance map.

After, the user hits the ‘plot’ button in the second screen the Compliance map is plotted with in the frequency range of interest entered as input. The compliance map is shown in the *Figure 2.3*.

3 Start and run the program at the first time

Caution, to be able to run the program MATLAB® version 6 or later version must be installed in the system.

The procedure is divided into the following steps

- First of all start MATLAB® and make sure that UFF files are available.
- Change the default or working directory to where this program is installed and see that all the UFF files are also located in the same folder. Type ‘Guistart’ in the command window to start the program.

- The first window contains a button at the bottom left corner push the button to enter the inputs.
- After entering the inputs in the second screen and hitting the “plot” button the result plotting compliance map opens in a separate window.

4 List of MATLAB® functions supplied by Saven Edutech®

anybutt.m

uitext.m

radiobut.m

editbox.m

fullscrn.m

animate.m

animate2.m

geodef.m

modeplot.m

cvfrfa2v.m

cvfrfd2v.m

univread.m

uf2frf.m

animate.m

animate2.m

animcalc.m

complexp.m

htoeplitz.m

poleadd.m

impresp.



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